

# Gravitation-induced quantum interference for neutrons

Dr. ir. ing. Victor de Haan

May 23-24, 2019

**BonPhysics Research and Investigations BV,  
Puttershoek, The Netherlands  
[www.bonphysics.nl](http://www.bonphysics.nl)**

Workshop on Matter-Wave Interferometry. May 23-24, 2019



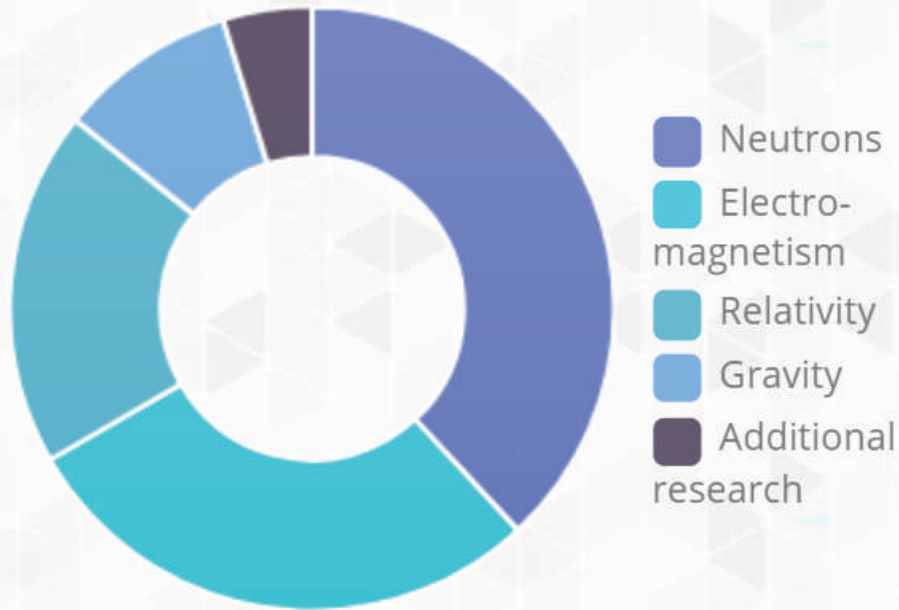
# Outline

- Introduction
- Principle
- Past measurements
- Future measurement
- Closure

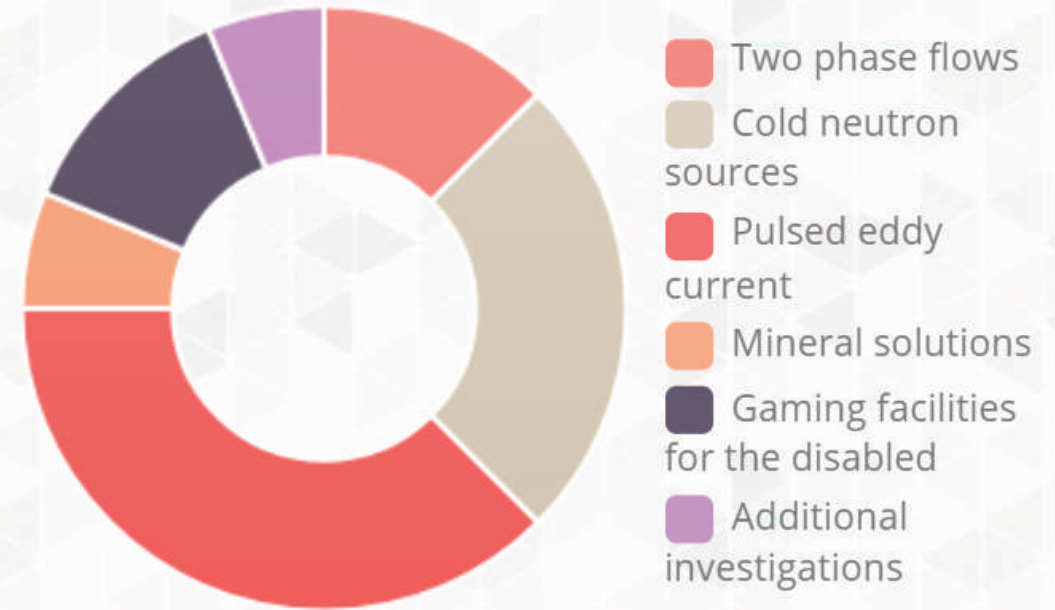
# Introduction

- Established: April 1, 1997
- 2 Employees
- Turnover ~100kEuro
- Consultancy and research

# Research



# Investigations

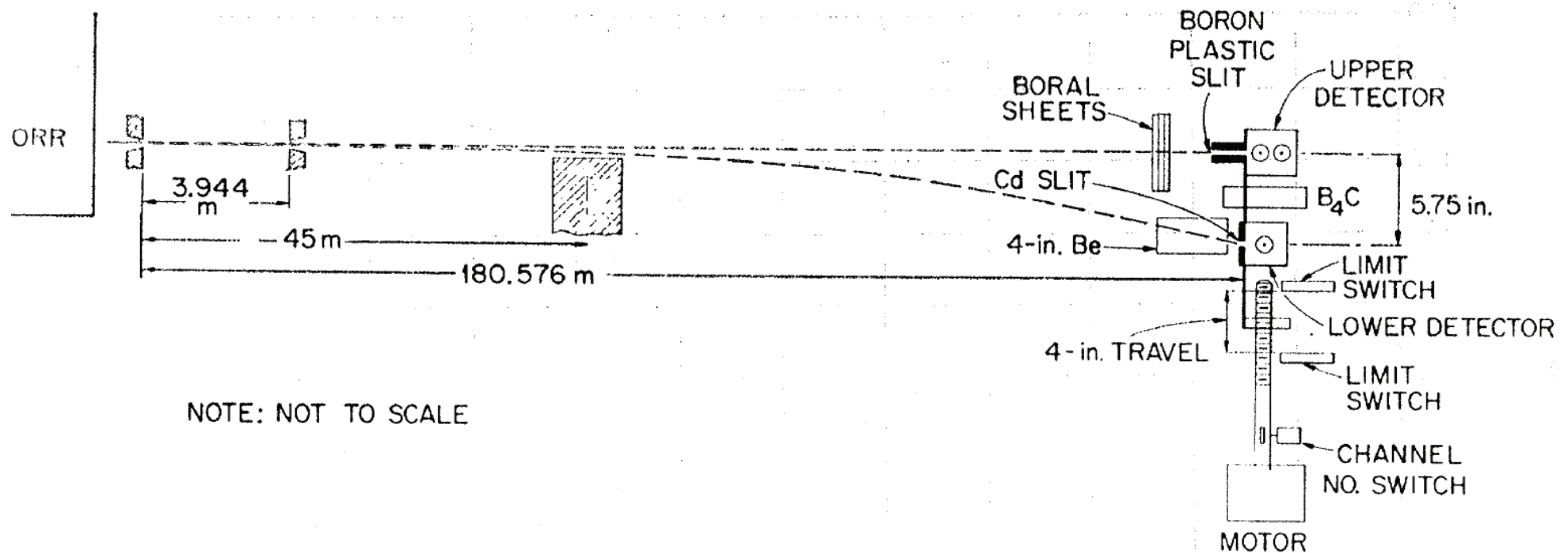


# Principle

- Neutrons fall down
- Coherence theory
- Interaction of neutron with magnetic field
- Propagation of polarization
- Spin echo

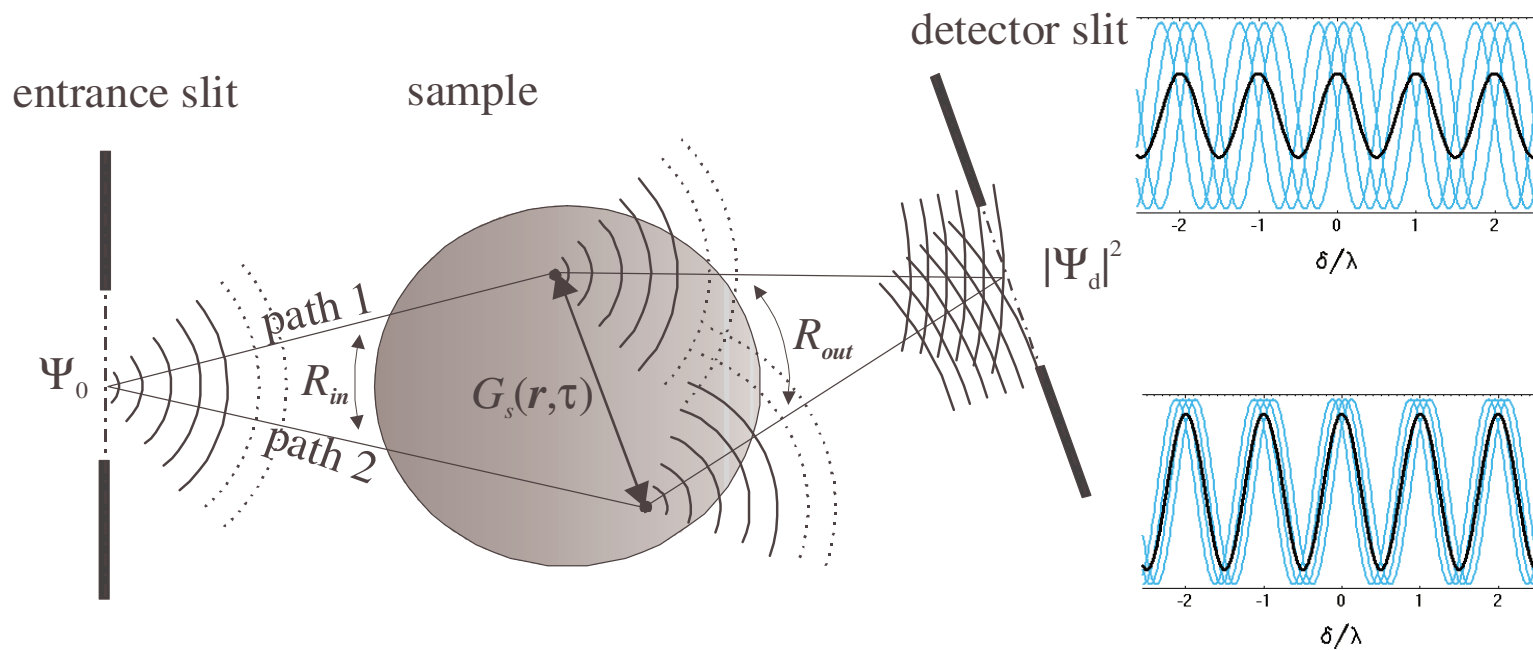
# Principle

- Neutrons fall down: McReynolds 1951, Dabbs 1965



# Principle

- Coherence theory: Scattering as coherence phenomena, Gähler et al. (1998)



# Principle

- Coherence theory: Mutual coherence function (Mandel, 1965)

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \langle \psi^*(\mathbf{r}_1, t) \psi(\mathbf{r}_2, t + \tau) \rangle_t$$

$$\Gamma(\mathbf{r}_1, \mathbf{r}_1, 0) = \langle \psi^*(\mathbf{r}_1, t) \psi(\mathbf{r}_1, t + 0) \rangle_t = J(\mathbf{r}_1)$$

$$I_d = 2v_p \int_{A_d} J(\mathbf{r}_d) d^2 r_d$$



# Principle

- Interaction of neutron with magnetic field

The Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}, t) \Psi(\mathbf{r}, t)$$

$$V(\mathbf{r}) = -\boldsymbol{\mu}_n \cdot \mathbf{B}$$

- potential energy due to spin: **Zeeman energy**
- include spinor description
- for the moment we omit interaction with material

# Principle

- Interaction of neutron with magnetic field

The potential energy of the neutron in a magnetic field and resulting kinetic energy are

$$E_{\text{pot}} = \hbar\omega_z = \pm\mu_n B; \quad E_{\text{kin}}^{\pm} = E_0 \mp \mu_n B; \quad k^{\pm} = k_0 \mp \Delta k,$$

with

$$\Delta k = \frac{m}{\hbar^2 k_0} \mu_n B = \frac{m\omega_z}{\hbar k_0} = \frac{\omega_z}{v_0} = \frac{-m\gamma_n}{2h} \lambda_0 B.$$

resulting in the Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial y^2} + \begin{bmatrix} \mu_n B & 0 \\ 0 & -\mu_n B \end{bmatrix} \Psi$$

# Principle

- Coherence function becomes coherence matrix

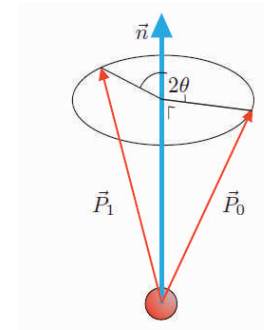
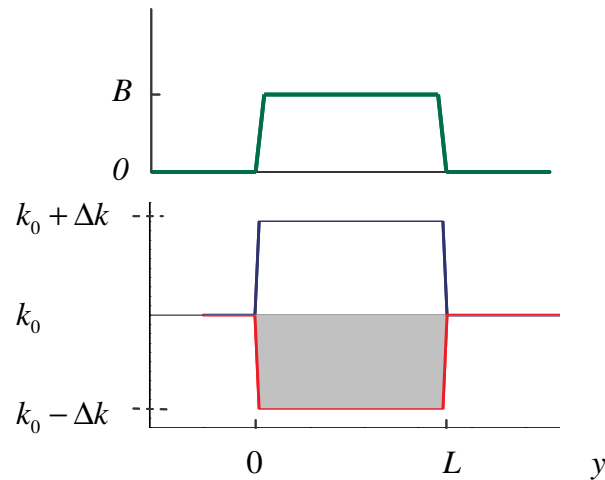
$$\hat{\Gamma}(\vec{r}_1, \vec{r}_2, \tau) = \left\langle \left( \begin{array}{c} \Psi^+(\vec{r}_2, t + \tau) \\ \Psi^-(\vec{r}_2, t + \tau) \end{array} \right) \left( \begin{array}{cc} \Psi^+(\vec{r}_1, t)^* & \Psi^-(\vec{r}_1, t)^* \end{array} \right) \right\rangle_t,$$

Coherence approach to  
neutron propagation  
in spin echo instruments

V.-O. de Haan

# Principle

- Interaction of neutron with magnetic field



Interpretation: the neutron spin (expect.value) precesses by an angle  $2\theta = 2\Delta kL = -m\gamma_n\lambda_0BL/h$  around the field direction, being the integration of the wave-number difference between the plus and minus wave function integrated over  $L$

$$p_x(L) = \langle \hat{\sigma}_x \rangle = ab^* + a^*b = \exp(-2i\Delta kL) + \exp(2i\Delta kL) = \cos 2\Delta kL$$

$$p_y(L) = \langle \hat{\sigma}_y \rangle = i(ab^* - a^*b) = i(\exp(-2i\Delta kL) - \exp(2i\Delta kL)) = \sin 2\Delta kL$$

# Principle

- Propagation of polarization

Mezei (1980): Polarisation is coherence between spinstates

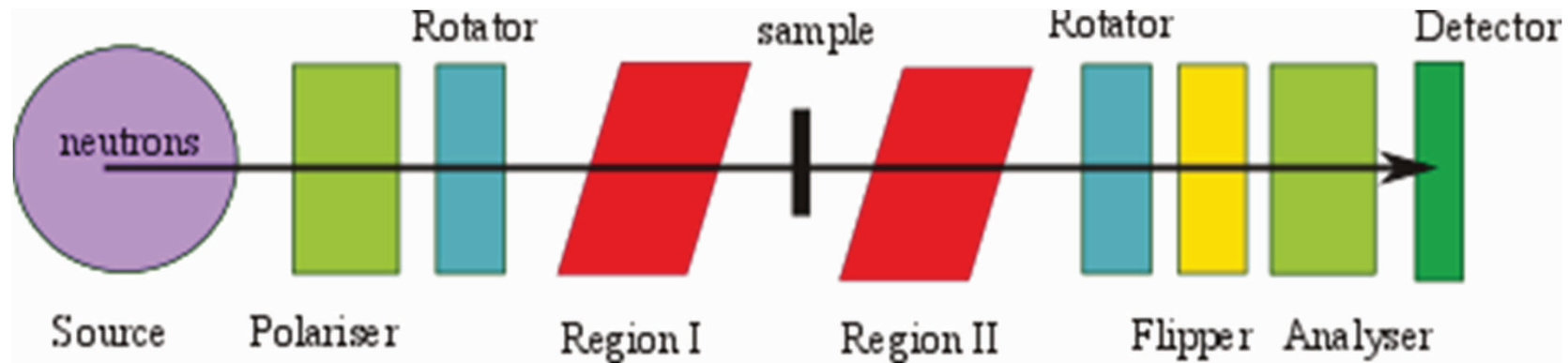
$$P_j(\vec{r}) = \frac{\left\langle \left( \Psi^+(\vec{r}, t)^* \quad \Psi^-(\vec{r}, t)^* \right) \hat{\sigma}_j \begin{pmatrix} \Psi^+(\vec{r}, t) \\ \Psi^-(\vec{r}, t) \end{pmatrix} \right\rangle_t}{\langle \Psi^+(\vec{r}, t) \Psi^+(\vec{r}, t)^* \rangle_t + \langle \Psi^-(\vec{r}, t) \Psi^-(\vec{r}, t)^* \rangle_t},$$

$$\hat{\Gamma}(\vec{r}_1, \vec{r}_2, \tau) = \left\langle \begin{pmatrix} \Psi^+(\vec{r}_2, t + \tau) \\ \Psi^-(\vec{r}_2, t + \tau) \end{pmatrix} \begin{pmatrix} \Psi^+(\vec{r}_1, t)^* & \Psi^-(\vec{r}_1, t)^* \end{pmatrix} \right\rangle_t,$$

$$P_j(\vec{r}) = \frac{\text{Tr}(\hat{\Gamma}(\vec{r}, \vec{r}, 0) \hat{\sigma}_j)}{\text{Tr}(\hat{\Gamma}(\vec{r}, \vec{r}, 0))},$$

# Principle to practice

- Propagation of polarization



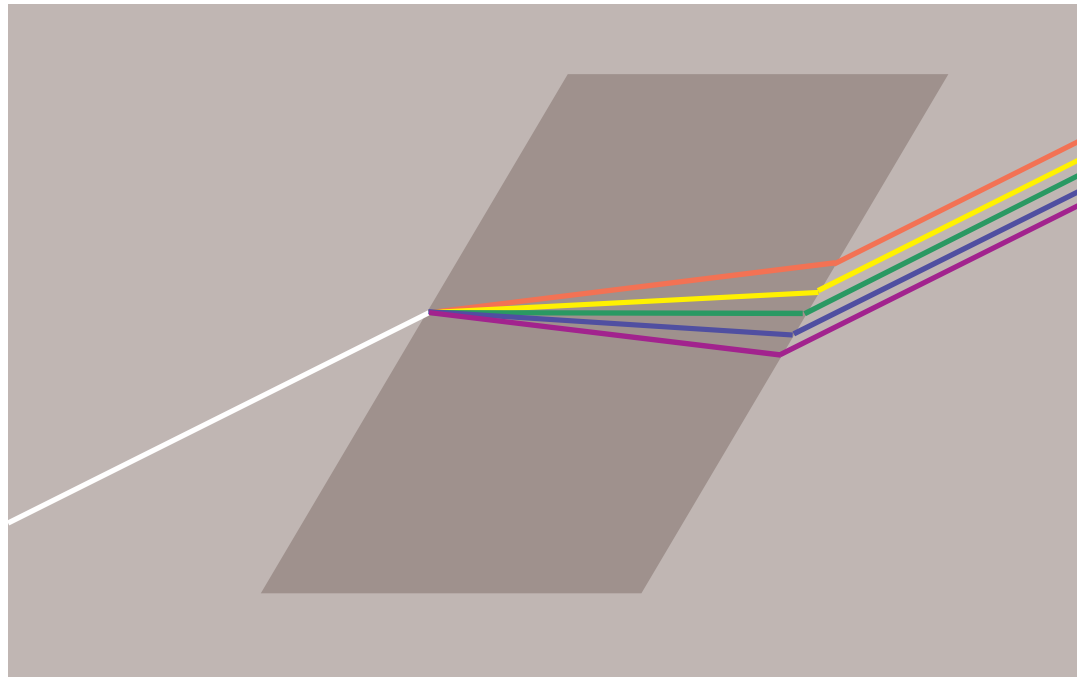
$$J_s^{(2)}(\vec{r}_d) = \frac{|\vec{J}_n(\vec{r}_d) + \vec{J}_f(\vec{r}_d)|}{2},$$

$$J_f^{(2)}(\vec{r}_d) = \frac{|\vec{J}_n(\vec{r}_d) - \vec{J}_f(\vec{r}_d)|}{2}$$

$$P_m^{(2)}(\vec{r}_d) = \frac{J_f^{(2)}(\vec{r}_d)}{J_s^{(2)}(\vec{r}_d)},$$

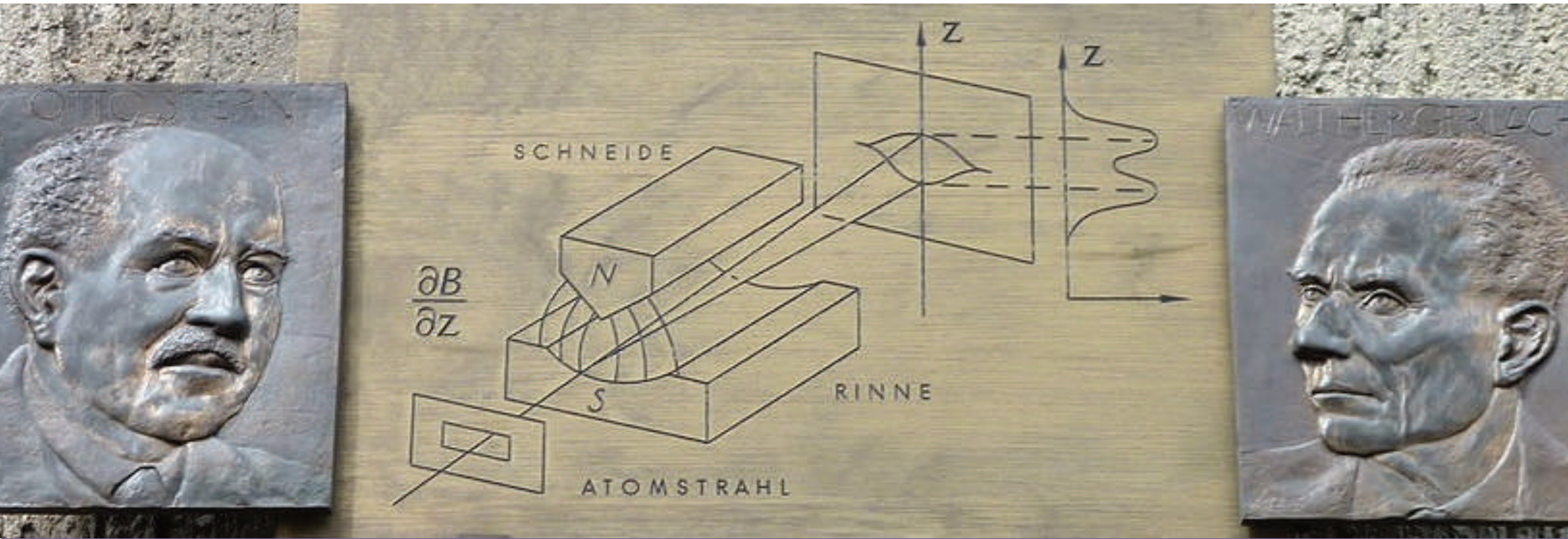
# Principle

- Refraction of light



# Principle

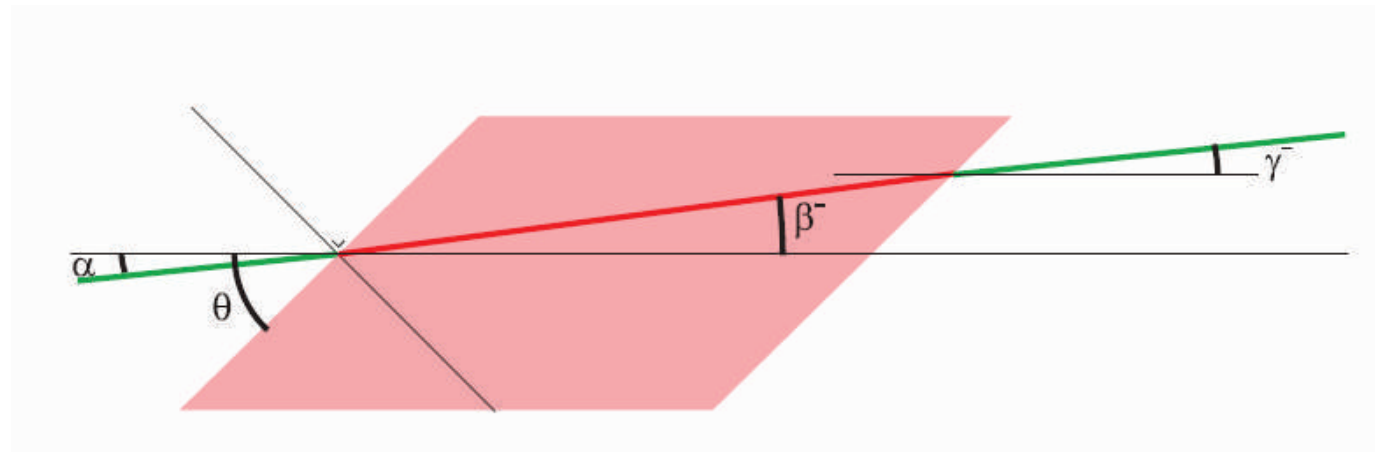
- Refraction at magnetic boundaries





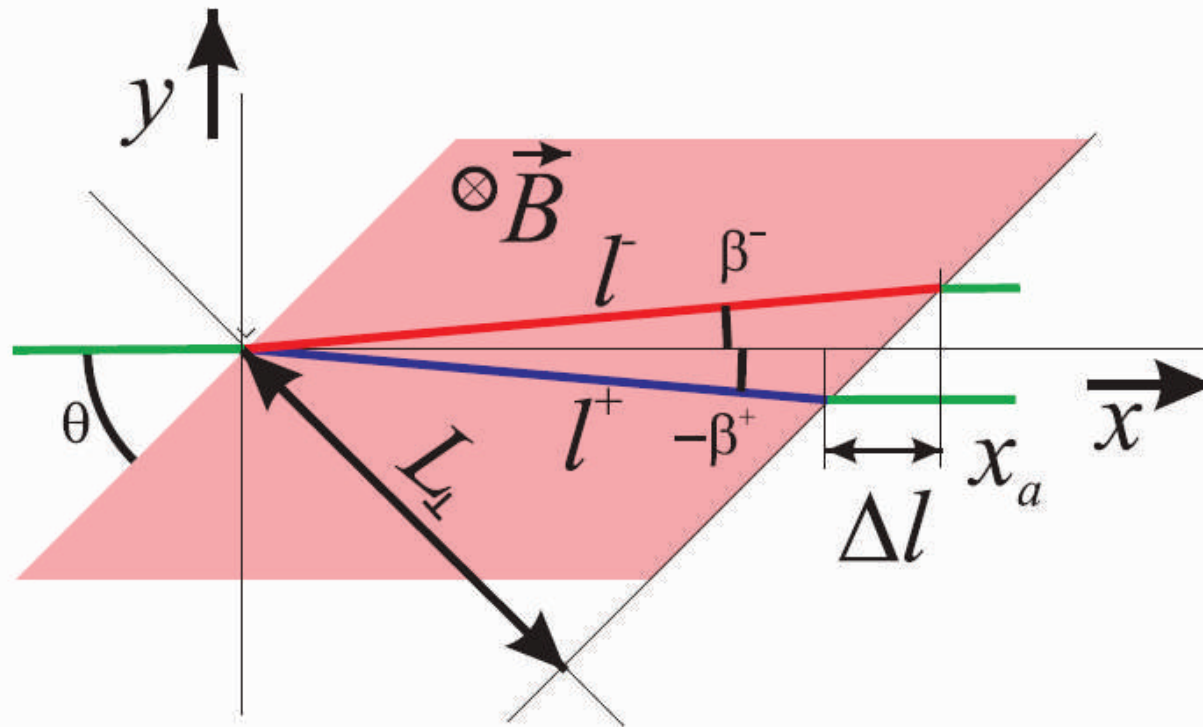
# Principle

- Refraction at magnetic boundaries



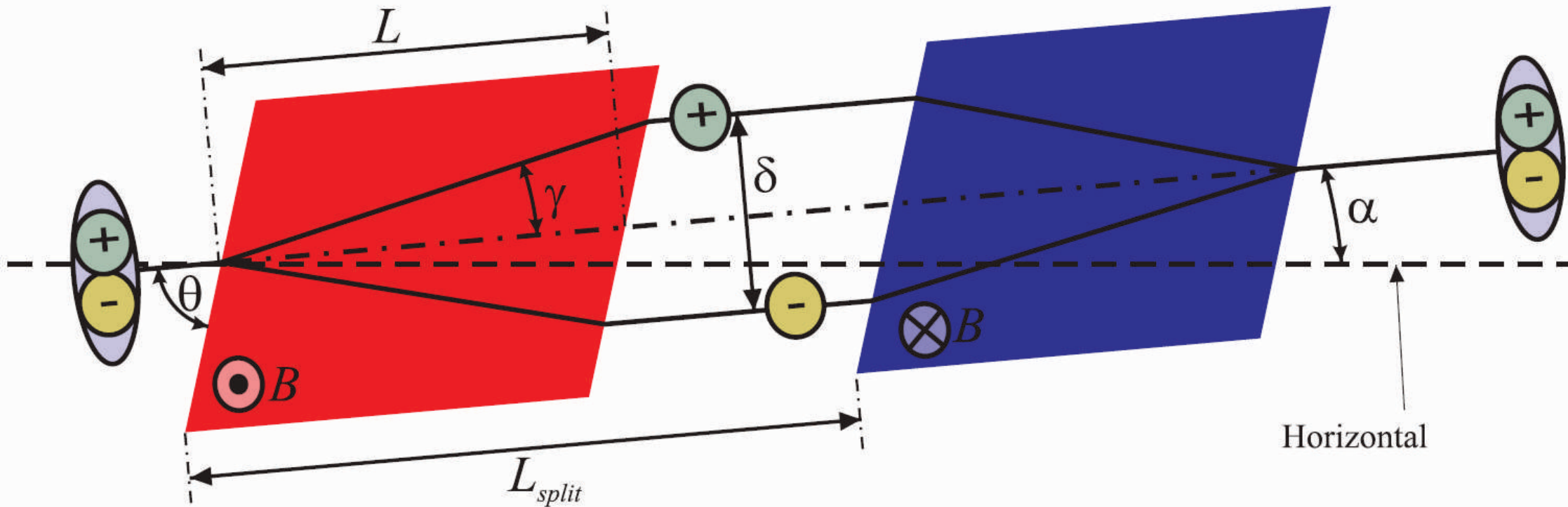
# Principle

- Refraction at magnetic boundaries



# Principle

- Spin Echo: Interferometer for the two spin states

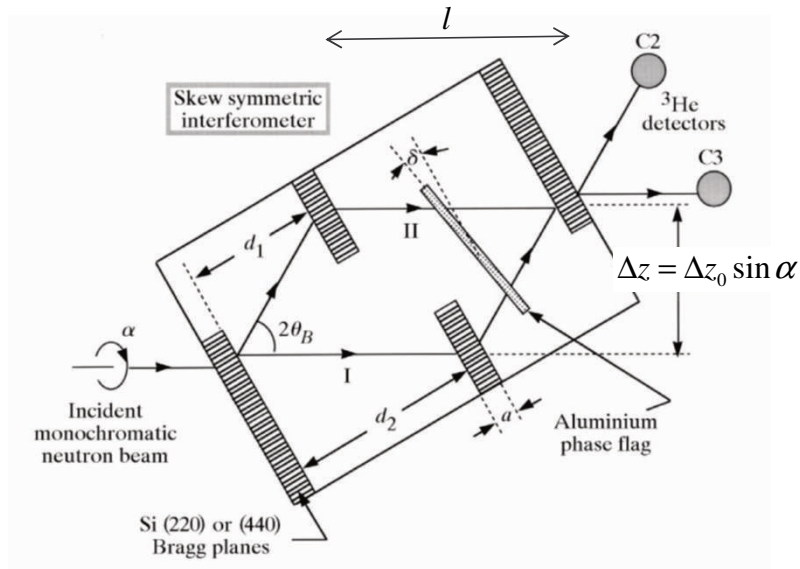


# Past measurements

- COW
- De Haan et al.

# Past Measurements

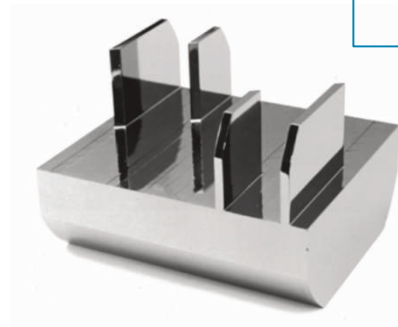
- Si single-crystal interferometer (COW experiments)



R. Colella *et al.*, Phys. Rev. Lett. **34** (1975) 1472  
 K.C. Littrell *et al.*, Acta Cryst. **54** (1998) 563

phase difference between both paths:

$$\Delta\phi = k_I l - k_{II} l \propto \Delta k_g l \Delta z$$

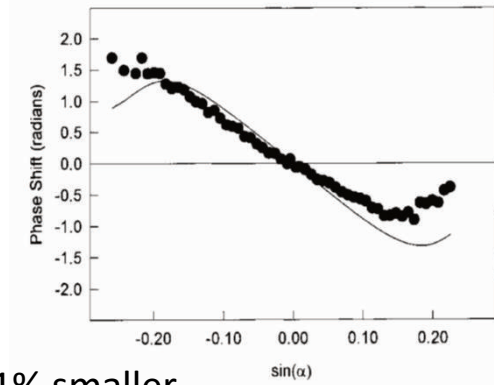


# Past Measurements

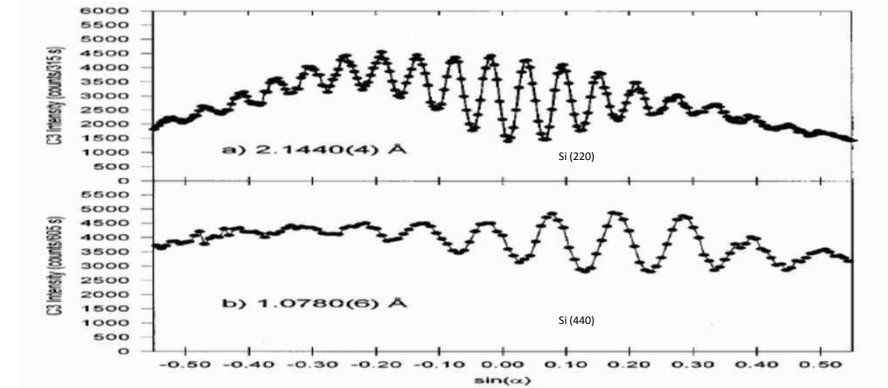
- Si single-crystal interferometer (COW experiments)

**Results:** interference signal as a function of extra phase added to both arms for two wavelengths

phase shift  $\Delta\phi$  as a function of rotation angle  $\alpha$



**Conclusion:** experimental phase shift is  $1.0 \pm 0.1\%$  smaller, compared with theory, when taking  $m_i = m_g$



some numbers

$$E_{220} = 18 \text{ meV}$$

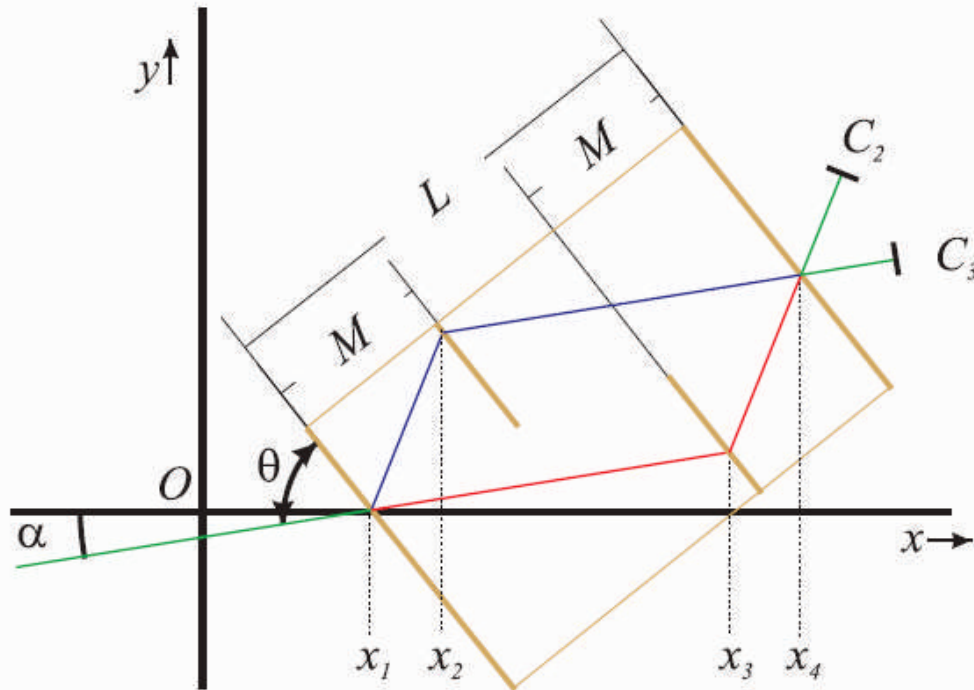
$$E_{440} = 70 \text{ meV}$$

$$\Delta z = 18 \text{ mm}$$

$$\Delta E_g \approx 2 \text{ neV}$$

# Past Measurements

- Si single-crystal interferometer (COW experiments)

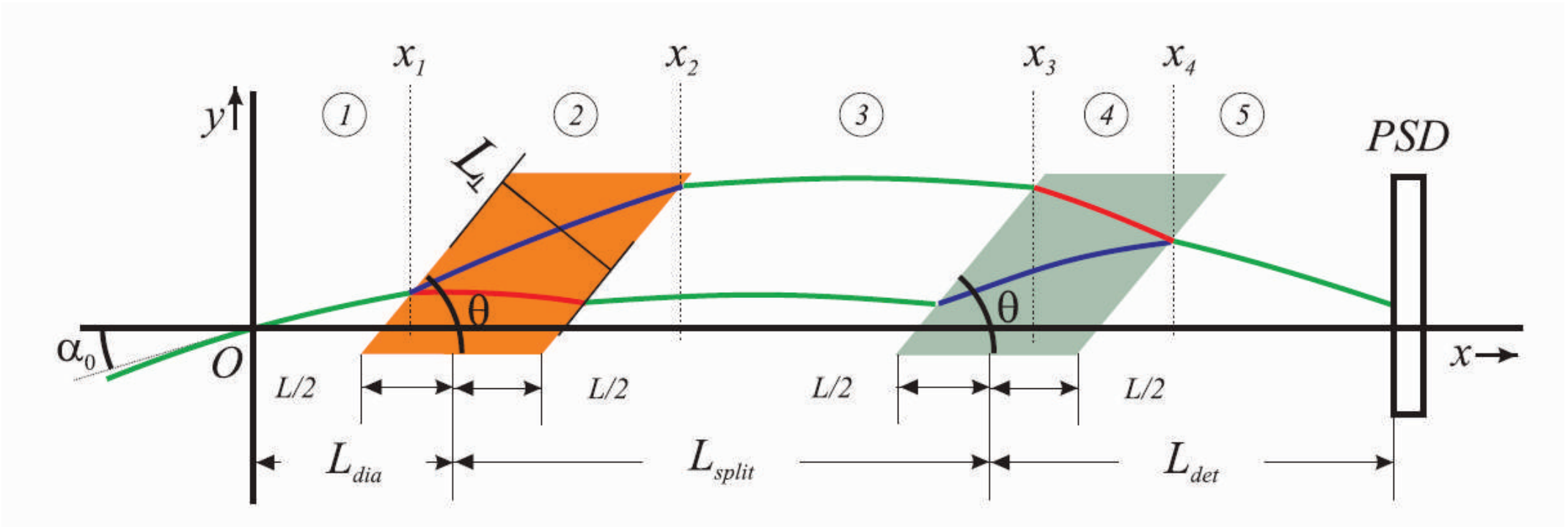


$$\frac{Am^2g}{\hbar^2k_0} \cos \alpha (1 - \tan \theta_B \tan \alpha),$$

## Conclusion:

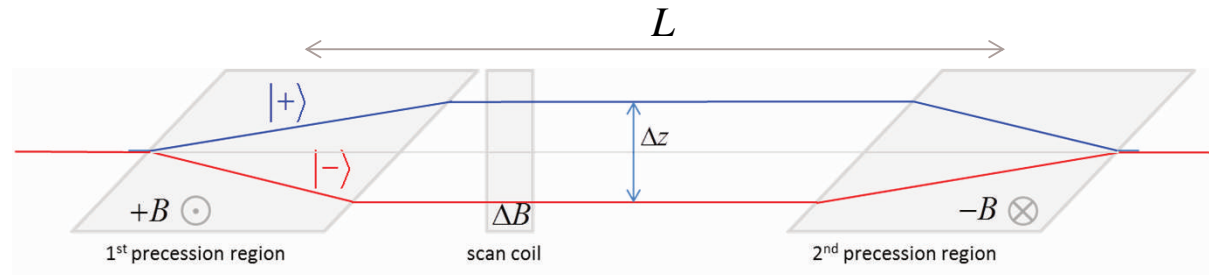
experimental phase shift is  $1.0 \pm 0.1\%$  smaller, compared with theory, when taking  $m_i = m_g$

# Past Measurements



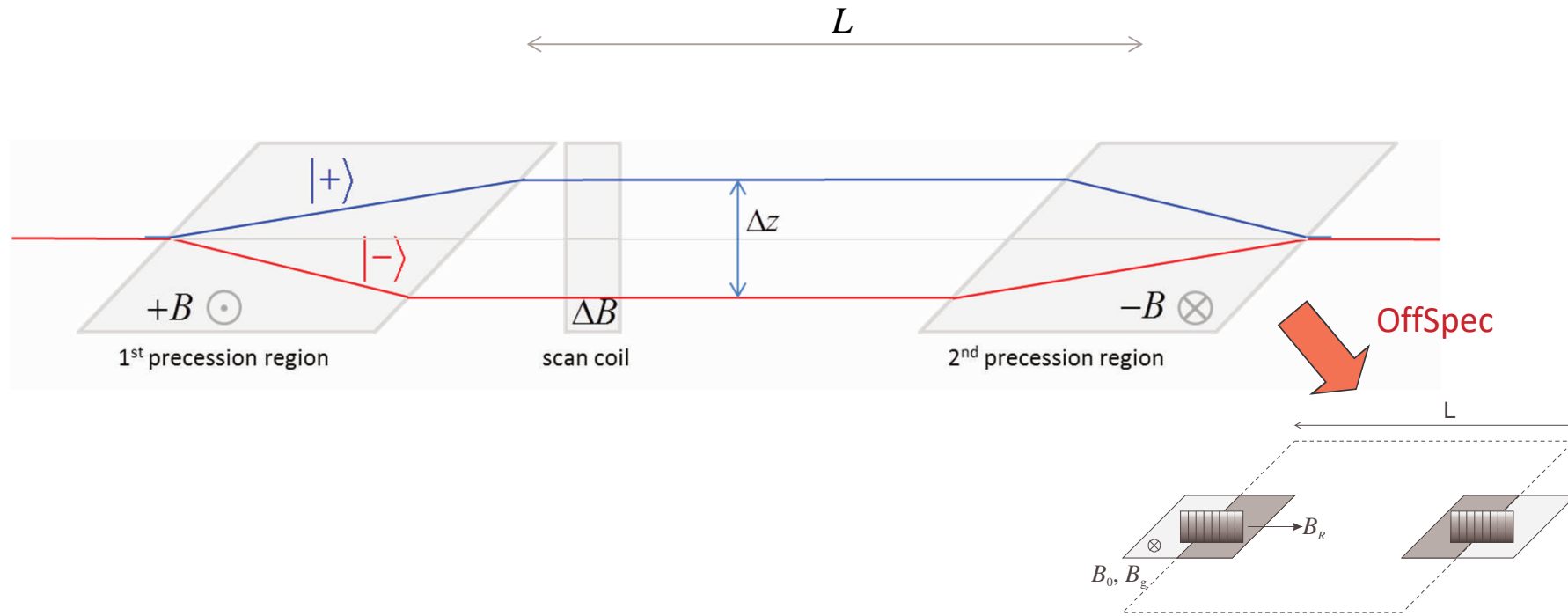


# Past Measurements

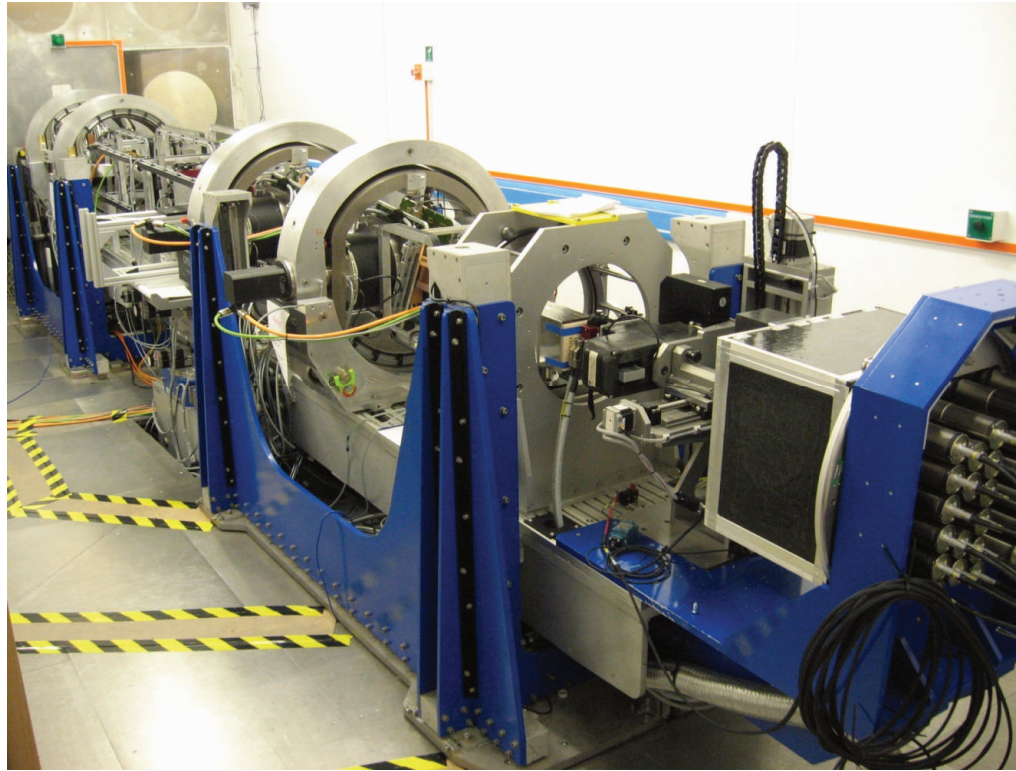


- Offspec is a time-of-flight instrument covering wavelength range  $0.2 < \lambda < 1.4$  nm
- The spin-echo polarisation of the recombined neutrons is measured:  $P(B, \lambda) = \langle \cos(\phi_L) \rangle$
- with Larmor phase: 
$$\phi_L = \int (k^+(x) - k^-(x)) dx \propto \Delta k_g L \Delta z$$
- extra phase is created by scan coil
- experiments are performed with splitting both in horizontal and vertical plane
- inclination angle of whole setup between -1.0 and +0.5 degrees

# Past Measurements



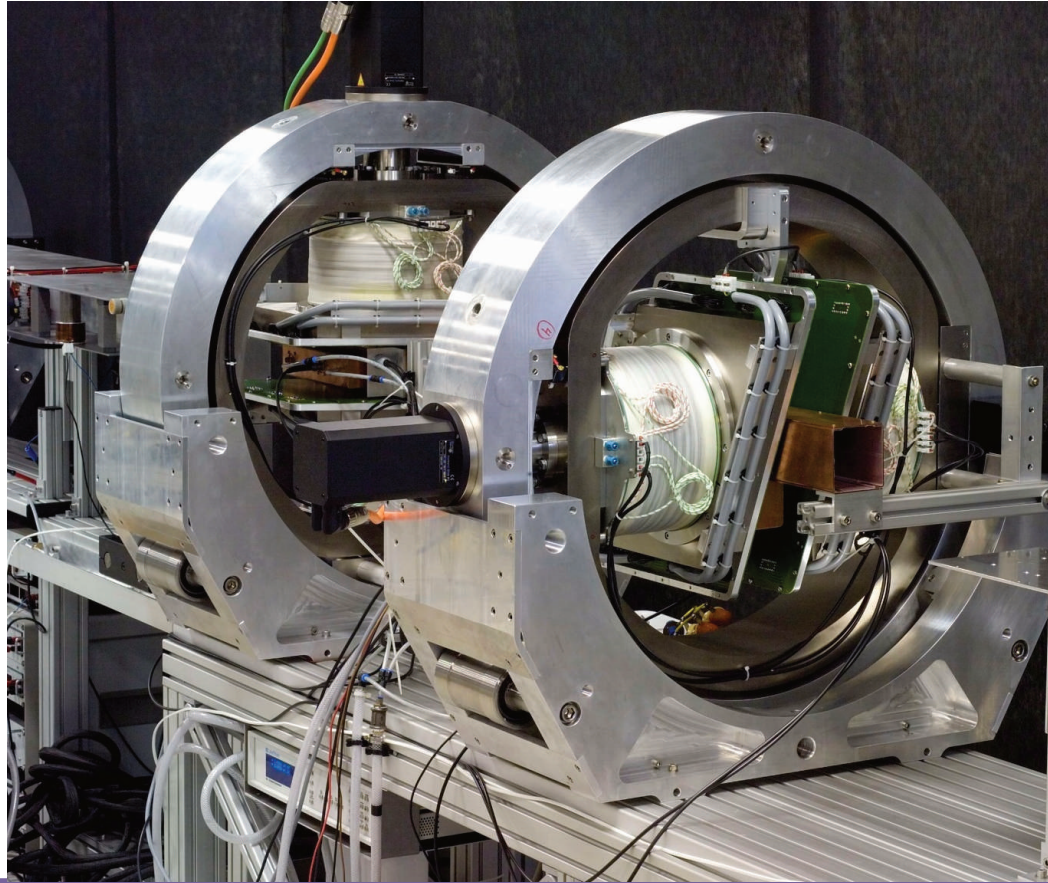
# Past Measurements



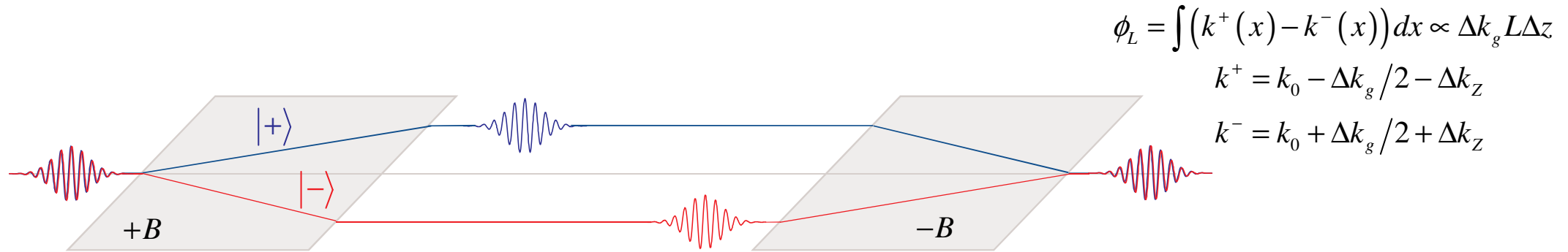
OffSpec @ ISIS

Reflectometer with spin-echo option  
meant for SERGIS and SESANS.

# Past Measurements

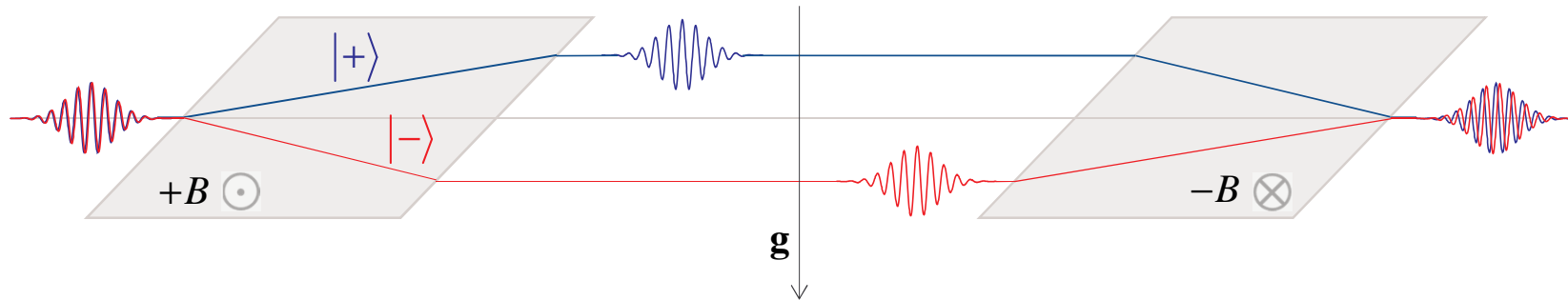


# Past Measurements



splitting in horizontal plane

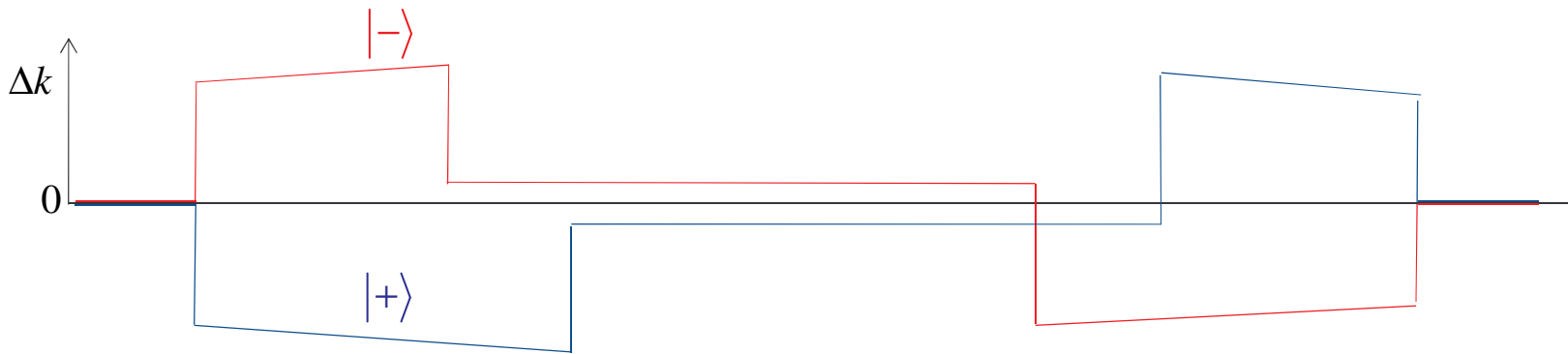
# Past Measurements



$$\phi_L = \int (k^+(x) - k^-(x)) dx \propto \Delta k_g L \Delta z$$

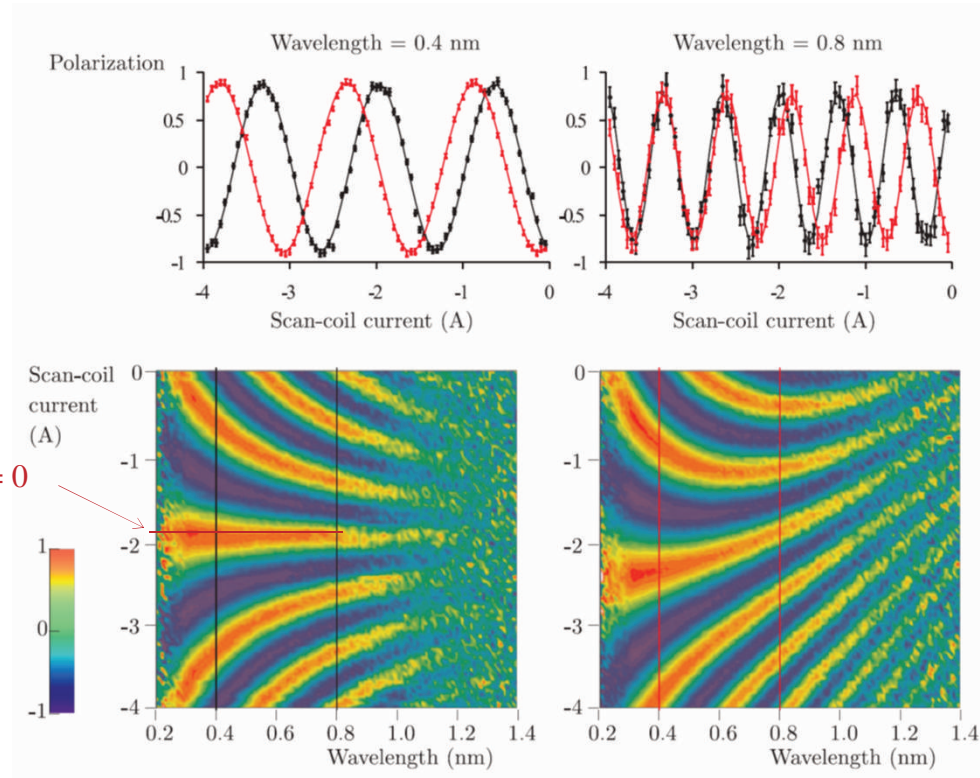
$$k^+ = k_0 - \Delta k_g / 2 - \Delta k_z$$

$$k^- = k_0 + \Delta k_g / 2 + \Delta k_z$$



splitting in vertical plane

# Past Measurements



**Result 1:** Contour plot of the spin-echo polarisation as a function of wavelength and extra phase added to both arms

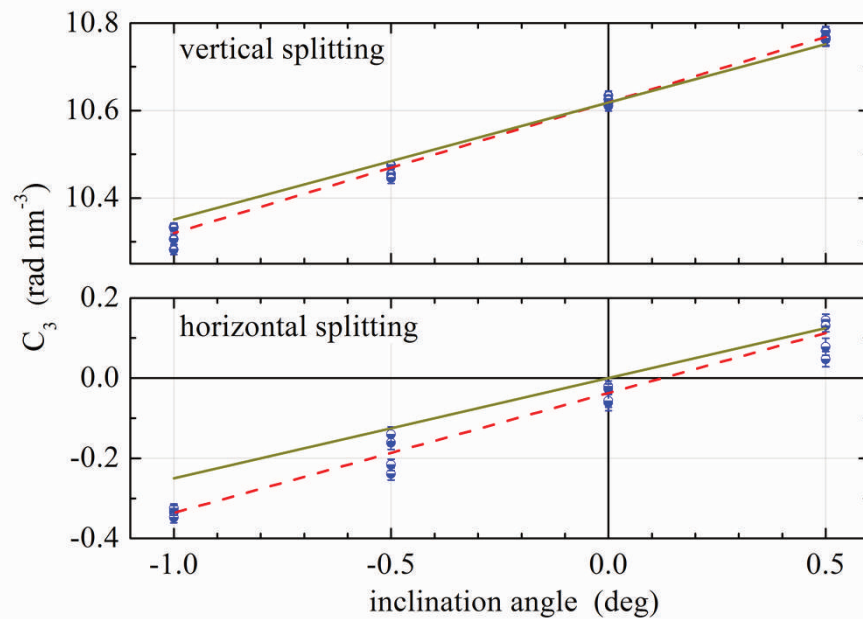
splitting in horizontal / vertical plane

# Past Measurements

Phases are described by  $\Delta\phi_L = C_1\lambda + C_2\lambda^2 + C_3\lambda^3$

$C_1$ : echo-condition  
 $C_2$ : Sagnac effect  
 (small correction)  
 $C_3$ : result of gravity

**Result 2:** Parameter  $C_3$  as a function of inclination angle



— theoretical value for  $m_i = m_g$   
 - - - best fit

## Conclusion:

experimental phase shift is, within the experimental accuracy of 0.1%, in agreement with theory, when taking  $m_i = m_g$

intercept  $C_3$ : exp:10.619(9) theory: 10.618(24)

## some numbers

$E_0 = 0.4 - 20$  meV  
 $\Delta E_z = 20$  neV  
 $\Delta z = 0.3 - 14$   $\mu\text{m}$   
 $\Delta E_g = 0.03 - 1.4$  peV



# Faraday, diary entry: March 19, 1849

What in gravity answers to the dual or antithetical nature of the forms of Force in Electricity and Magnetism?

Perhaps the *to* and *fro*, that is the ceding to the force or approach of Gravitating bodies and the effectual reversion of the force of separation of the bodies, quiescence being the neutral condition.

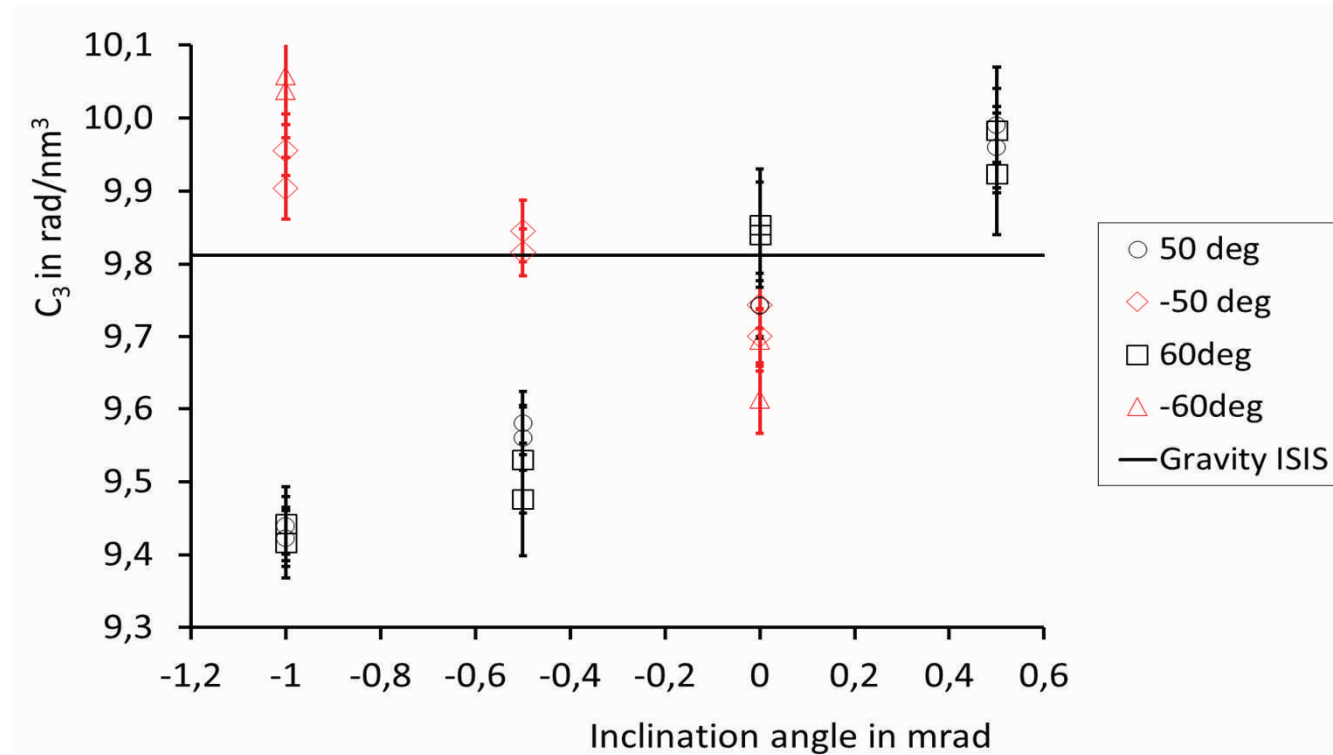
Try the question experimentally on these grounds – then the following suppositions or suggestions arise ...

Bodies approaching by gravitation, and bodies separated per force, whilst gravitating towards each other, may shew in themselves or in surrounding matter or helices, *opposite currents* of electricity round line of motions as an axis. But if not moving *to* or *fro* each other, should produce no effect.

# Faraday, diary entry: March 19, 1849

All this is a dream. Still, examine it by a few experiments.

# Future measurement



# Future measurement

- Spin-Gravity Coupling at OffSpec – July 2019
- Follow project on research gate:  
<https://www.researchgate.net/project/Spin-Gravity-coupling>

# Future measurement

$$H = \frac{k}{c} \vec{g} \cdot \vec{S} \quad \text{or} \quad H = \frac{k \hbar}{c 2} \sigma \cdot \vec{g}$$

# Future measurement

$$H = \frac{k}{c} \vec{g} \cdot \vec{S} \quad \text{or} \quad H = \frac{k \hbar}{c 2} \sigma \cdot \vec{g}$$

$$H = -\frac{\hbar}{2} \sigma \cdot \gamma \vec{B} \quad \vec{B}_{eff} = \vec{B} - \frac{k}{\gamma c} \vec{g}$$

Hence, the effect cannot be independently measured.

# Future measurement

$$H(\vec{r}) = H_{VA}(\vec{r}) + H_{AA}(\vec{r})$$

$$H_{VA}(\vec{r}) = S_{VA} \frac{\lambda_o}{\lambda} g_V g_A \sigma \cdot \frac{\vec{v}}{v} \eta_{VA} \quad S_{VA} = \frac{M}{m} H_o \frac{\lambda_o}{R} = 1.21 \cdot 10^{32} \text{ eV}$$

$$H_{AA}(\vec{r}) = S_{AA} \frac{\lambda_o}{\lambda} (g_A)^2 \sigma \cdot \left( \frac{\vec{v}}{v} \times \vec{e}_z \right) \eta_{AA} \quad S_{AA} = \frac{M}{m} H_o \alpha \left( \frac{\lambda_o}{R} \right)^2 = 5.3 \cdot 10^8 \text{ eV}$$

yielding a possible limit on  $g_V g_A$  of  $10^{-48}$  and on  $(g_A)^2$  of  $5 \cdot 10^{-24}$

# Closure

In the coherence approach picture each neutron spectrometer can be seen as a neutron interferometer.

In the coherence approach picture the polarisation of a neutron beam can be interpreted as the interference between two coupled wave functions.

Neutron spin-echo methods can be effectively used to study fundamental physics questions.



# Thanks for your attention!

Dr. ir. ing. Victor de Haan

May 24-24, 2019

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41

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